Optimal Monetary Policy in an Economy with Industry and Services

Milan Lisicky

Preliminary

November 22, 2012

Abstract

What is the optimal measure of inflation in an economy with two sectors that differ in the share of labour and capital used in production? By inspecting a micro-founded social welfare function of the monetary authority, this paper finds that the optimal measure of inflation in this model economy is systematically biased toward the capital-intensive sector. The source of this bias is traced back to a different slope of the marginal product of labour in the two sectors. Our findings speak to the literature that studies economies with multiple sectors differentiated by nominal and demand-side characteristics. The contribution of this paper is that it turns attention to characteristics behind the supply side of the economy and inspects their role for the design of the optimal policy.

1 Introduction

This paper investigates what measure of inflation should a welfare-minded central bank target in an economy with nominal price and wage rigidities and two sectors that differ in the share of labour used in production. The welfare-theoretic analysis of monetary policy had traditionally focused at reducing the 'shoe-leather' costs associated with the opportunity costs of holding money. Following the publication of Rotemberg and Woodford (1997), researchers started to study also other sources of frictions, most notably those brought about by delays in the adjustment of nominal prices contracts. For instance, Woodford (2003) presents a framework in which asynchronous price adjustment that leads to discrepancies between relative prices causes an inefficient allocation of resources. A major advantage of this approach compared to the traditional one is that it provides specific guidelines to the central bank regarding the measure of inflation and/or output gap to target in order to maximise utility of economic agents.

The issue of an optimal target of a monetary authority has received a considerable interest in the literature. In a multi-sector or multi-country setting, Aoki (2001) proved that for an economy where one sector exhibits nominal frictions while the other sector's prices are perfectly flexible the optimal policy is to stabilise inflation in the sticky-price sector. In this particular framework, the monetary policy that follows only one of the two sectors also helps establish the efficient outcome for the entire economy. Aoki's work justifies the tendency of central banks to target the "core" inflation (an index that excludes goods whose prices change frequently) instead of targeting an overall measure of prices. Benigno (2004) extends this result to the open-economy setting and concludes that if the central bank can commit itself to inflation-targeting policies, it is optimal to give higher weight to the country/sector with greater nominal rigidities (see Proposition 4 in Benigno, 2004).

Erceg, Henderson and Levin (2000) and Erceg and Levin (2006) study the implications for monetary policy of nominal frictions in the labour market coupled with differences in the durability of output. The former paper shows that staggered price and wage contracts in a one-sector economy imply a trade-off between the goals of inflation and output gap stabilisation. Erceg and Levin (2006), which is the work most related to this paper, then implement the same features in a two-sector setting. The key lesson that emerges from their study is that factors unrelated to the source of nominal frictions (durability of output) can affect the design of optimal policy. The monetary authority in their model prefers to give unequal weight to the two sectors even though the nominal frictions are equally severe across sectors. In particular, the authors show that the optimal inflation target in this economy is biased toward the durable goods sector.

The aim of this paper is to investigate the monetary-policy implications of another important and highly realistic facet of the economic landscape, namely of the differences in the relative use of labour in the production process. My model economy is closed and consists of two sectors. For the sake of illustration, the relatively labour-intensive one is labelled 'services' while the relatively capital-intensive one is labelled 'manufacturing'. I abstract from all other differences between the two sectors. I show that a central bank operating under the inflation-targeting regime finds it optimal in this specific environment to target an inflation index that puts systematically higher weight on inflation in manufacturing. The result originates in different slopes of the marginal product of labour in the two sectors. The marginal product of labour is steeper in the capital-intensive sector, implying that a given change in output would require a greater percentage change in the labour input compared with the labour-intensive one.

The key assumptions that underlie this result are imperfect competition in the goods market and nominal price rigidities. Imperfect competition makes firms supply differentiated goods that are all purchased in the equilibrium. Nominal rigidities then create a non-degenerate dispersion in individual prices and outputs in both sectors and lead to inefficiencies that can be addressed by the toolkit of the monetary authority. Assuming goods weigh equally in terms of utility, note that a fictitious social planner entrusted by the task to produce a given level of the consumption basket would spread the production of individual goods equally among all firms. However, such an outcome is infeasible in a market economy if prices of the individual goods differ from each other. Limited substitutability then implies that there is more output (and therefore labour) needed to generate the same amount of utility compared with the equilibrium of the social planner. The faster the marginal product falls with the increase in output the more of the labour input is needed to produce the additional level of output and the greater are welfare losses compared with the efficient allocation. A welfare-maximizing central bank therefore strives to limit the dispersion in output of individual goods in manufacturing more emphatically than in services. It does so by reducing the incentives to alter prices after a shock hits the manufacturing sector, which substantiates greater weight of manufacturing inflation in the optimal inflation index.

In terms of the modelling approach, this paper builds on the analysis of Erceg et al. (2000), who extend the welfare-theoretical approach to monetary policy by allowing for the presence of two production factors, and generalizes it to consider the implications of different labour intensities. Another (and rather technical) point of departure from

Erceg et al. (2000) is that the analysis in the present paper is embedded in a framework with a utility function that is not separable in sectoral bundles; as was shown in Aoki (2001) and Benigno (2004) this feature is not only more realistic but also allows for richer implications for the conduct of monetary policy.

The paper is organized as follows. Section 2 introduces the economy and defines the equilibrium. Section 3 presents log-linear versions of the key model equations and solves for the equilibrium under flexible prices and wages. Finally, section 4 derives the social welfare function and discusses the optimal structure of the inflation index.

2 Model

The model is a variant of Rotemberg and Woodford (1997) with imperfect competition and Calvo-style (random duration) price contracts in goods markets. It follows Erceg and Levin's (2000) strategy to incorporate random-duration nominal wage contracts and production factors into the model. On top of that, factor intensities are allowed to differ across sectors. The utility function is such that the structural equations of the model depend on relative prices.

The economy consists of two sectors that differ in their relative use of labour and capital. Manufacturing is assumed to be the relatively capital-intensive sector and services the relatively labour-intensive one (note that this is only a rough description of reality, because a heavy use of capital is typical for some services too, for example in telecommunications). Both factors are free to move within a sector but are sufficiently specialized so that they cannot be used interchangeably in both sectors. Similarly as Erceg et al. (2000) and Erceg and Levin (2006), this model abstracts from endogenous capital accumulation. The following part of the section describes the behaviour of households and firms, respectively, in the model.

2.1 Households

The decision makers in this economy are two sets of identical infinitely-lived and fully forward-looking households, each of measure one, that supply their work in the labour markets in one of the two sectors. They consume an index of consumption goods produced in both sectors, which is represented by the Cobb-Douglas aggregator (1). The aggregator exhibits unitary elasticity of substitution between the sectoral bundles and the shares of sectoral consumption bundles are given by ψ_m and ψ_s, which add to one. Households allocate their expenditure in a cost-minimizing way according to prices of the respective sectoral bundles (2) so that the aggregate price index corresponding to one unit of their aggregate consumption is given by (3). Note that the household-specific subscripts are suppressed from reasons that will shortly become apparent.

$$C_t \equiv \frac{C_{m,t}^{\psi_m} C_{s,t}^{\psi_s}}{(\psi_m)^{\psi_m} (\psi_s)^{\psi_s}} \tag{1}$$

$$C_{j,t} = \left[\frac{P_{j,t}}{P_t}\right]^{-1} \psi_j \, C_t, \quad j \in \{m, s\} \tag{2}$$

$$P_t = P_{m,t}^{\psi_m} P_{s,t}^{\psi_s} \tag{3}$$

At the sectoral level, households consume all the differentiated goods produced in each sector but the degree of substitution between them, θ, is higher than the one between goods produced in different sectors. Their preferences over the continuum of goods are captured by the Dixit-Stiglitz utility function (4), in which each good carries equal weight. Similarly as above, optimality conditions imply demand functions (5) and the price index of the sectoral basket (6).

$$C_{j,t} \equiv \left[\int_0^1 C_{j,t}(f)^{\frac{\theta-1}{\theta}} \, df\right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \tag{4}$$

$$C_{j,t}(f) = \left[\frac{P_{j,t}(f)}{P_{j,t}}\right]^{-\theta} C_{j,t} \tag{5}$$

$$P_{j,t} = \left[\int_0^1 P_{j,t}(f)^{1-\theta} \, df\right]^{\frac{1}{1-\theta}} \tag{6}$$

Apart from consumption, each household has to determine the amount of work it wishes to supply in the labour market, over which it is endowed with monopolistic power; hours worked by different households are therefore imperfect substitutes. Each firm in the production process employs a labour index $L_{j,t}$ that is composed of hours worked by all households in the sector. It is useful to think of the labour aggregation process as if it was carried out by a fictitious labour agency that observes all the wage contracts in force and allocates workers to the composite labour index in a cost-minimizing fashion described by the demand functions (8). Subsequently, the agency passes the index at cost $W_{j,t}$, defined in (9), to all firms operating in the sector. The agency allows workers to adjust the wage contracts only after passing of a random interval of time (Calvo-style wage contacts).

$$L_{j,t} \equiv \left[\int_0^1 L_{j,t}\left(h\right)^{\frac{\phi-1}{\phi}} dh \right]^{\frac{\phi}{\phi-1}}, \quad \phi > 1 \tag{7}$$

$$L_{j,t}\left(h\right) = \left[\frac{W_{j,t}\left(h\right)}{W_{j,t}} \right]^{-\phi} L_{j,t} \tag{8}$$

$$W_{j,t} = \left[\int_0^1 W_{j,t}\left(h\right)^{1-\phi} dh \right]^{\frac{1}{1-\phi}} \tag{9}$$

With these preliminary definitions in mind we are now ready to define the households' decision problem. Their objective at time t is to maximise a time-separable utility function composed of the consumption index (1), the hours worked $L_{j,t}$ and money balances $M_{j,t}$, which provide liquidity services. The objective function takes the following form

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \mathrm{U}\left[C_{t+i}\left(h\right)\right] + \mathrm{V}\left[L_{j,t+i}\left(h\right)\right] + \mathrm{M}\left[\frac{M_{j,t+i}\left(h\right)}{P_{t+i}}\right] \right\} \tag{10}$$

where the operator E_t denotes expectations over all possible future states of nature conditional on the information available in t and β is a discount factor. Apart from nominal money balances, one-period state-contingent bonds $B_{t,s}$ are traded in the economy. They do not enter utility but serve as a risk sharing device. The price of a financial claim to one unit of nominal income in a particular state of nature s in the next period is $\delta_{t,t+1,s}$, which will be henceforth referred to as the stochastic discount factor. If the central bank in this economy supplies risk-less one-period bonds then it follows that the gross nominal interest rate on these assets at t satisfies $R_t = \left[\mathrm{E}_t \delta_{t,t+1}\right]^{-1}$.

The period budget constraint then imposes that household's h labour income and wealth at the beginning of period t must be spent either on consumption, lump-sum taxes or purchases of financial assets.

$$\begin{aligned} P_t C_t(h) + M_{j,t+1}(h) + \int_s \delta_{t,t+1,s} B_{t+1,s}(h) \\ \leq \left(1+g_w\right) W_{j,t}(h)\, L_{j,t}(h) + M_{j,t}(h) + B_t(h) + H_t(h) - T_t(h) \end{aligned} \tag{11}$$

where $H_t\left(h\right)$ denotes household's aliquot share in firms' profits and also includes the income from administering the fixed level of capital in that sector. Note that the role of government in this economy is limited to the collection of lump-sum taxes that are spent on price-subsidies to firms g_p and wage subsidies to households g_w. The idea behind these subsidies is to make the central bank solely responsible for distortions that originates from nominal rigidities, while the government applies transfer schemes to eliminate the inefficiencies related to monopolistic markets.

The first-order conditions of this problem with respect to consumption and bonds are given by

$$\mathrm{U_C}\left[C_t\left(h\right)\right] = \Lambda_t P_t \tag{12}$$

$$R_t^{-1} = \beta \mathrm{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \tag{13}$$

Assuming that the central bank carries out its monetary policy by setting the nominal interest rate, Rotemberg and Woodford (1997) show that it is safe to neglect the optimality condition for real money balances. The additional first-order condition would only pin down the nominal level of money. Due to the assumption of complete markets, the marginal utility of income, Λ_t, is common to all households so that they choose an identical level of consumption.

Finally, let us turn attention to the decision problem of households adjusting their wage contracts in period t. Since the labour agency allows to re-optimise the nominal wage only to a fraction $(1-\eta)$ of randomly selected workers each period, an individual worker therefore takes into account that he or she might not be able to change the wage in the subsequent periods. The worker will thus maximise the utility function (10) with a discount factor $(\beta\eta)$, which reflects the expected length of the new wage contract, subject to the budget constraints (11) and the labour demands (8), which are taken into account because the worker is a monopolistic provider of $L\left(h\right)$. Assuming that the government provides the worker with subsidy g_w that eliminates the monopolistic distortions, the first order condition with respect to wages is given by

$$\mathrm{E_t} \sum_{i=0}^{\infty} \left(\beta\,\eta\right)^i W_{j,t+i}^{\phi} L_{j,t+i} \left\{ \mathrm{V_L} \left[\left(\frac{W_{j,t}(h)}{W_{j,t+i}} \right)^{-\phi} L_{j,t+i} \right] + \Lambda_{t+i} W_{j,t}(h) \right\} = 0 \tag{14}$$

Because this equation is independent of all worker-specific variables but wages it follows that all workers adjusting their wage in a given period will set them at an identical level, $W_{j,t}^*$, which greatly simplifies the sectoral wage index (9). It can be shown (as in Calvo, 1983) that it becomes

$$W_{j,t}^{1-\phi} = \eta W_{j,t-1}^{1-\phi} + \left(1-\eta\right) W_{j,t}^{*\,1-\phi} \tag{15}$$

Workers, who are unable to adjust wages in the given period, commit themselves in the wage contract to provide their variety of labour at the wage rate set in preceding periods according to the labour demands (8). Note that under flexible prices, the optimality conditions for consumption (12) and labour (14) imply that wages must equal to the marginal rate of substitution, which is defined here for future reference:

$$MRS_{j,t} = -\frac{V_L(L_{j,t})}{U_C(C_t)} \tag{16}$$

2.2 Firms

Each individual firm is a monopolistic supplier of the differentiated good $Y_{j,t}(f)$, which it produces by means of the Cobb-Douglas technology (17) using capital services and the index of labour hours described above. In order to investigate the implications of different labour intensity for the conduct of monetary policy, labour intensity is allowed to differ across sectors with $\alpha_m \geq \alpha_s$. I further assume that all firms operating in a sector are subject to common shocks that jointly affect the productivity of both production factors in the sector. The production function is defined by

$$Y_{j,t}(f) = A_{j,t} K_{j,t}(f)^{\alpha_j} L_{j,t}(f)^{1-\alpha_j} \tag{17}$$

Similarly to households, firms sign Calvo-style nominal price contracts that allow them to change their prices only at exogenous random intervals. Specifically, a fraction $(1-\epsilon)$ of firms gets to choose a new price at the beginning of a quarter. The probability of re-adjusting the price in any given period is independent of time that has elapsed since the firm set its price the last time. The decision problem of firm f therefore consists of maximizing the present discounted value of expected profits achieved under the assumption that the price set in t would, with probability ϵ, apply also in the future, subject to the demand functions for its particular variety given by (2) and (5). Taking into account the market clearing conditions $Y_{j,t} = 2\,C_{j,t}$ and assuming that the government applies production subsidy g_p to offset monopolistic mark-ups, the first-order condition of this problem is

$$\mathrm{E_t} \sum_{i=0}^{\infty} \epsilon^i \delta_{t,t+i} \left(\frac{P_{j,t}(f)}{P_{j,t+i}} \right)^{-\theta} \left(\frac{P_{j,t+i}}{P_{t+i}} \right)^{-1} Y_{t+i} \left(P_{j,t}(f) - MC_{j,t+i} \right) = 0 \tag{18}$$

Note that in the limiting case of fully flexible prices when $\epsilon = 0$ this equation imposes that prices equal the cost $MC_{j,t}$ arising from production of one unit of $Y_{j,t}(f)$.

Firms not able to re-set their prices in the period solve the following minimisation problem

$$\min_{K,L} P^k_{j,t} K_{j,t}(f) + W_{j,t} L_{j,t}(f) \quad \text{s.t.} \quad A_{j,t} K_{j,t}(f)^{\alpha_j} L_{j,t}(f)^{1-\alpha_j} \geq 1 \tag{19}$$

the first-order conditions of which imply, first, that all firms in a sector employ identical capital-labour ratios and, secondly, that marginal costs can be expressed as a weighted average of factor prices divided by the productivity shock $A_{j,t}$ or, alternatively, as a ratio of wages and the marginal product of labour.

$$MC_{j,t} = \frac{P^{k,\alpha_j}_{j,t} W^{1-\alpha_j}_{j,t}}{A_{j,t}\, \alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}} = \frac{W_{j,t}}{(1-\alpha_j)\, A_{j,t} K_j^{\alpha_j} L_{j,t}^{-\alpha_j}} = \frac{W_{j,t}}{MPL_{j,t}} \tag{20}$$

Writing marginal costs in terms of wages and the marginal product of labour simplifies the solution of the model because it allows to abstract from computing the price of capital (note that the sectoral level of capital is fixed). Since marginal costs are independent of firm-specific variables, it follows from the price setting equation (18) that all firms adjusting their prices in a given period will choose the same price $P^*_{j,t}$. The sectoral price index (6) therefore simplifies to

$$P_{j,t}^{1-\theta} = \epsilon P_{j,t-1}^{1-\theta} + (1-\epsilon) P_{j,t}^{*\,1-\theta} \tag{21}$$

The last equation states that prices in force in period t can be decomposed into a fraction ϵ of price contracts passed from the previous period and a fraction $(1-\epsilon)$ of newly adjusted prices. To complete the description of the block of firms it remains to define the sectoral level of output that corresponds to the sectoral labour index $L_{j,t}$, used in the equations above, and the fixed level of capital in a sector. Canzoneri, Cumby and Diba (2005) show that it is given by

$$Y_{j,t} = A_{j,t} K_j^{\alpha_j} L_{j,t}^{1-\alpha_j} (DP_{j,t})^{-1} \tag{22}$$

$$DP_{j,t} \equiv \int_0^1 \left[\frac{P_{j,t}(f)}{P_{j,t}} \right]^{-\theta} df$$

where the last term reflects price dispersion in sector j – a term, that will play a crucial role in the analysis below. Its inverse relationship to sectoral output aptly illustrates how price dispersion decreases the amount of output available for consumption for a given level of the labour index. Canzoneri at al. (2005) relate the price dispersion to welfare losses by observing that the social planner, in order to maximise consumption, would allocate production of differentiated goods equally to all firms (because their weights in the consumption index are equal). But since firms, due to nominal price rigidities, charge different prices in the equilibrium, the Pareto efficient allocation is infeasible. Lower demands for one good must be more than compensated by higher demands for other goods (due to their imperfect substitutability) to achieve a given level of utility. Higher sectoral output then implies higher demand for labour and correspondingly greater disutility from labour. Analogously to the price and wage indices defined above, it can be shown that the price dispersion can be restated in a computationally simpler form of

$$DP_{j,t} = \epsilon \left(\frac{P_{j,t-1}}{P_{j,t}} \right)^{-\theta} DP_{j,t-1} + (1-\epsilon) \left(\frac{P^*_{j,t}}{P_{j,t}} \right)^{-\theta} \tag{23}$$

2.3 Equilibrium and parameterisation

The preceding subsections described the optimal behaviour of households and firms and their mutual relationships. We are now ready to define the equilibrium for the whole economy. It is defined as a set of allocations that includes the aggregate and sectoral consumption bundles C_t and $C_{j,t}$ and the labour indices $L_{j,t}$, and a set of prices that includes the aggregate and sectoral price indices P_t, $P_{j,t}$, wage indices $W_{j,t}$, prices and wages of those who can adjust them in the given period, $P^*_{j,t}$ and $W^*_{j,t}$, and the marginal costs $MC_{j,t}$ so that, for a given level of the nominal interest rate R_t set by the central bank and the stochastic productivity processes (defined below), the following conditions hold for all $t \geq 0$:

1. households maximise consumption over time according to (12) and (13) and set wages according to (14)
2. firms set prices according to (18) and (20)
3. the price and wage indices satisfy (3), (21) and (15)
4. sectoral outputs are given by the demand functions (2)
5. sectoral labour indexes follow from the aggregate resource constraints as stated in (22) and (23)
6. markets clear, so that we have $Y_{j,t} = 2\,C_{j,t}$ for both sectors and $Y_t = 2\,C_t$ at the aggregate level.

This system does not have a closed-form solution, nonetheless it can be identified by perturbing the deterministic equilibrium, in which the price and wage inflation is zero and all variables take constant values over time. Description of how this system behaves in a vicinity of the deterministic equilibrium is a topic of section 3.2. Before turning to the log-linear analysis though, a description of the functional forms and parameter values used in the computations is due.

The following functions for the period utility functions of consumption and leisure are assumed in the model:

$$\mathrm{U}(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$$

$$\mathrm{V}_j(L_{j,t}) = \kappa_j \frac{(1-L_{j,t})^{1-\chi}}{1-\chi}$$

Allowing the leisure preference parameter κ_j to differ across sectors, as e.g. in Erceg and Levin (2006), greatly facilitates the derivation of the social welfare function because it permits to consider the steady-state elasticity of substitution in leisure identical across sectors. Different elasticities would bring about an additional source of dissimilarity, which would obscure the results presented in this paper. For the sake of comparability, the above-referred study is also the source of most of the parameters used here, in particular of σ, χ, θ and ϕ. The structural parameters in the utility functions are set to $\sigma = 2$ and $\chi = 3$ and the shares of labour hours to leisure in the steady state are equal to $1/2$ in both sectors, which calibrates the weights of leisure in the utility function κ_j. Next, in order to simplify interpretation of optimal targeting rules, the shares of sectors in the aggregate output are equal to $\psi_m = \psi_s = 0.5$. The discount rate β equals 0.99 implying that the steady-state real interest rate is 1.01% on a quarterly basis or roughly 4% annually. Parameters $\theta = \phi = 4$ so that steady state mark-ups are equal to 33%. Finally, the expected contractual duration of prices and wages is four quarters (the parameters ϵ and η are both equal to 0.75, as in Erceg et al., 2000).

What remains to specify are the properties of stochastic innovations. Erceg and Levin (2006) characterise them by means of bivariate AR(1) processes, which allows the coefficients and standard errors to differ in both sectors. However, since the strategy followed in this paper is to isolate the effects of different labour intensities, the properties of shocks are assumed to be symmetric here. They follow a bivariate first-order stochastic process $A_t = 0.95\, A_{t-1} + e_t$, where e_t is an *i.i.d.* process with variances $\sigma_m^2 = \sigma_s^2 = 0.0086^2$, in line with the traditional RBC literature, and $corr\,(e_m, e_s) = 0.29$, as in Erceg and Levin (2006). The computational approach followed here is based on the estimation of variances of certain variables included in the model, so only the properties of stochastic processes matter, the particular draws of shocks are not relevant for the results.

3 Equilibrium of the log-linearised model

The linear-quadratic approach of Rotemberg and Woodford (1997) to welfare evaluation relies on the second-order Taylor-series approximation to the social welfare function and first-order approximations to the structural equations of the model that were derived in the previous section. This section outlines the structural equations in their log-linear form while the derivation of the social welfare function is postponed to section 4. The structural equations shown here define how the system responds to *small* shocks that perturb the non-stochastic equilibrium (small in the sense that first-order Taylor series expansion still provides an accurate description of the system).

This section is divided into two parts: the first solves for the equilibrium of the log-linear model in the case of fully flexible wages and prices, which represents the Pareto efficient allocation in this model. The other section derives the equations in the presence of nominal price and wage contracts[1].

3.1 Flexible price and wage equilibrium

Let us start with defining the two fundamental parameters of the log-linear model: elasticity of marginal utility of consumption and elasticity of marginal disutility of labour. The former is defined as $\rho = -\frac{U_{CC}}{U_C}\overline{C}$ and the latter as $\omega = \frac{V_{LL}}{V_L}\overline{L}$. With the functional forms specified above it is evident that these parameters are common to both sectors and are equal to σ and $\chi/2$, respectively. The weighted average of shocks in the economy is $a_t^w = \psi_m a_{m,t} + \psi_s a_{s,t}$ and, finally, the common denominator of the equations that follow, Λ, is equal to $\omega + \rho + (1-\rho)(\psi_m \alpha_m + \psi_s \alpha_s)$. With these definitions in mind, we can now proceed to characterise the solution of the efficient equilibrium.

The real interest rate, as defined by the Euler equation (12, 13), is given by $i_t^n = \mathrm{E}_t \rho \left(y_{t+1}^n - y_t^n \right)$. Socially optimal allocation requires that the impacts of productivity shocks are spread equally among households in both sectors, so that their consumption (and the aggregate output) corresponds to

$$y_t^n = \left((1+\omega)/\Lambda \right) a_t^w \tag{24}$$

Similarly, the social planner then distributes the given change in production equally among all workers, therefore also the labour indices (and individual hours of work) respond only to the economy-wide average of shocks

$$l_{j,t}^n = \left((1-\rho)/\Lambda \right) a_t^w$$

These two equations imply that the marginal rate of substitution is equalized across sectors, hence also real wages (normalized by the aggregate price level) respond identically:

[1] In what follows, all nominal variables are rendered stationary by suitable transformations so that wages and prices set in a given period are standardized by the corresponding sectoral indices. The sectoral price and wage aggregates and the nominal interest rate are divided by the aggregate price level. A percentage deviation of a variable X_t from its steady state value $\overline{X}$ will be denoted with lower-case letters, e.g. x_t, and represent a first-order approximation to $ln\left(X_t/\overline{X}\right)$. Superscripted variables, x^n, denote the value of the variable x in the efficient equilibrium and, finally, $\hat{x}$ stands for the gap between the actual level of a variable and its value in the efficient equilibrium, $x - x^n$.

$mrs^n_{j,t} = w^n_{j,t} = ((\omega + \rho)/\Lambda)\ a^w_t$. However, the social planner recognizes that the marginal product of labour, defined in (20), is steeper in the capital-intensive sector and allocates the sectoral outputs accordingly.

$$mp\,l^n_{j,t} = a_{j,t} - \alpha_j\,((1-\rho)/\Lambda)\,a^w_t \tag{25}$$

$$y^n_{j,t} = (1-\alpha_j)\,((1-\rho)/\Lambda)\ a^w_t + a_{j,t} \tag{26}$$

At this place it is appropriate to foreshadow that the steeper slope of the marginal product in manufacturing will, with rigid prices, motivate the desire of the central bank to fight inflation in the manufacturing sector more emphatically than in services because a given change of output will require greater response of the labour index there. We will return to this point in section 4. For future reference, relative prices are defined as $p_{rel,t} = p_{m,t} - p_{s,t}$ and reflect the different response of sectoral outputs:

$$p^n_{rel,t} = (\alpha_m - \alpha_s)\,((1-\rho)/\Lambda)\ a^w_t - (a_{m,t} - a_{s,t}) \tag{27}$$

This concludes the description of the efficient equilibrium of the model and we now turn to the case of asynchronous price and wage adjustments.

3.2 Equilibrium with nominal rigidities

The demand side of the model is characterized by the inter-temporal IS equation

$$\hat{y}_t = \mathrm{E}_t\hat{y}_{t+1} - \frac{1}{\rho}\,(i_t - \mathrm{E}_t\pi_{t+1} - i^n_t) \tag{28}$$

derived from the first-order conditions of the household problem. This equation embodies the negative dependence of the output gap, $\hat{y}$ on the gap between the real interest rate and its counterpart in the efficient equilibrium, where the latter summarizes how current and expected future productivity shocks affect the course of the economy. Some authors refer to it from this reason as the “Wicksellian” natural rate of interest (e.g., Woodford, 2003). By solving the equation forward one can further show that the central bank can stabilise the *aggregate* output gap by adjusting the expected path of the real interest rate along the natural interest rate.

The supply block of the dynamic model consists of two sets of behavioural equations that characterise the price setting and wage setting processes, respectively, and several definitions describing the linkages between sectoral price and wage indices.

First, log-linearising the firms' price setting equation, (18), together with the sectoral price index, (21), yields

$$\pi_{j,t} = \mu_p\,(w_{j,t} - mp\,l_{j,t} - p_{j,t}) + \mathrm{E}_t\beta\pi_{j,t+1} \tag{29}$$

where the parameter $\mu_p = \frac{1-\epsilon}{\epsilon}(1-\epsilon\beta)$ determines the degree of inflation persistence. The marginal product of labour follows from the definition in (20), where the labour index is replaced using the sectoral resource constraint (22). Substituting in the solutions from the efficient equilibrium and re-arranging (as in Erceg et al., 2000), it is possible to show that the marginal product is negatively related to the sectoral output gap (the last term at the right-hand side). To conserve space, the following expression uses signed shares of sectoral consumption bundles in the consumption aggregator $\widetilde{\psi}_m = \psi_m$ and $\widetilde{\psi}_s = -\psi_s$.

$$mp\,l_{j,t} = w^n_{j,t} - p^n_{j,t} - \frac{\alpha_j}{1-\alpha_j}\left(\hat{y}_t - \widetilde{\psi}_j\,\hat{p}_{rel,t}\right) \tag{30}$$

By following a similar strategy, from the wage-setting relationship (14) and the wage index (15) it is possible to derive a dynamic equation that determines sectoral wage inflation as a function of expected wage inflation and the deviations of the marginal rate of substitution from the real wage.

$$\pi^w_{j,t} = \mu_w\left(mrs_{j,t} - w_{j,t}\right) + \mathrm{E}_t\beta\pi^w_{j,t+1} \tag{31}$$

where the coefficient of inertia in wage inflation $\mu_w = \frac{1-\eta}{\eta}(1+\omega\phi)(1-\eta\beta)$ plays a similar role as μ_p above. The marginal rate of substitution is obtained by log-linearising (16)

$$mrs_{j,t} = w^n_{j,t} + \left(\frac{\omega}{1-\alpha_j} + \rho\right)\hat{y}_t - \widetilde{\psi}_j\left(\frac{\omega}{1-\alpha_j}\right)\hat{p}_{rel,t} \tag{32}$$

where one can observe that lower substitutability in consumption (higher ρ) makes the marginal rate of substitution relatively more responsive to the aggregate output gap than to changes in relative prices. This will become important in section 4, where it will counteract the central bank's willingness to allow extensive deviations in outputs across sectors.

Finally, the following relationships between prices, wages and the corresponding inflation rates close the system of log-linear equations:

$$\pi_{j,t} = p_{j,t} + \pi_t - p_{j,t-1} \tag{33}$$

$$\pi^w_{j,t} = w_{j,t} + \pi_t - w_{j,t-1} \tag{34}$$

$$\hat{p}_{rel,t} = p_{m,t} - p_{s,t} - p^n_{rel,t} \tag{35}$$

$$0 = \psi_m p_{m,t} + \psi_s p_{s,t} \tag{36}$$

where the first two equations simply follow from the definitions of inflation rates (normalized by the aggregate price index), the third equation defines the relative price gap and the final relationship is a log-linear version of the aggregate price index (3). Assuming that the path of the nominal interest rate is exogenously determined by the central bank, the equations reported in this section form a block of fourteen linear equations that fully describe the dynamic response of all endogenous variables to productivity shocks.

4 Nominal rigidities and welfare

This section turns to the main question addressed in this paper: what is the optimal policy of the central bank when sectors do not employ identical factor proportions and how can it be implemented? To answer the first question, a social welfare function is derived to see what variables should the central bank stabilize, while the issue of its successful implementation will be discussed subsequently by means of numerical simulations.

I follow here the work of Rotemberg and Woodford (1997), who showed how to derive a social welfare function from microeconomic foundations and how to compare it to its ideal value achievable in an economy without nominal frictions. Furthermore, the derivation of the welfare criterion consistent with this particular model makes use of strategies outlined in Aoki (2001), Benigno (2004) and Erceg et al. (2000). To start with, the social welfare is defined as an unconditional expected value of the sum of households' discounted utility. Taking unconditional expectations is needed in order to take into account all possible histories of shocks that may have occurred prior to date t. Before taking the unconditional expectations, the sum of discounted (*time t-conditional*) utilities is denoted SW_t

$$SW_t = E_t \sum_{i=0}^{\infty} \beta^i \left[2\, U(C_{t+i}) + \int_0^1 V(L_{m,t+i}\,(h))\, dh + \int_0^1 V(L_{s,t+i}\,(h))\, dh \right] \tag{37}$$

The criterion that the monetary authority wishes to maximise is defined as $SW = E\,(SW_t - SW_t^n)$. The law of iterated expectations then guaranties that this problem is equivalent to minimizing the *period* average utility losses from deviating from the efficient equilibrium.

The Rotemberg and Woodford's method proceeds by taking second-order Taylor expansions of each element in (37), which allows one to describe the welfare criterion exclusively as a function of second moments of the aggregate and sectoral output gaps and the price and wage inflation rates in each sector[2].

$$E\,\widehat{SW} = \gamma_y\,(1-\rho)\,var\,\hat{y} - \psi_m\,\gamma_y\,\frac{1+\omega}{1-\alpha_m}\,var\,\hat{y}_m - \psi_s\,\gamma_y\,\frac{1+\omega}{1-\alpha_s}\,var\,\hat{y}_s - \frac{\gamma_p}{1-\alpha_m}\,var\,\pi_m - \frac{\gamma_p}{1-\alpha_s}\,var\,\pi_s - \gamma_w\,var\,\pi_m^w - \gamma_w\,var\,\pi_s^w \quad (38)$$

plus terms independent of policy and the third- and higher-order terms. This welfare function implies that there are two kinds of variables to be taken into account by the social planner – the gaps between the actual and the optimal levels of output and sectoral measures of nominal variability – both of which are rooted in the presence of nominal rigidities but hint at different aspects of welfare losses[3].

The former term, variance of output gaps, reflects the costs of output deviations from its efficient level and points to the objective of the social planner to respond to sector-specific shocks so that resources are allocated efficiently across sectors. Note that the presence of output *gaps* indicates that a complete stabilisation of the level of output is undesirable; the central bank aims, instead, at establishing such changes in output that would prevail under flexible prices (recall that nominal rigidities are the only source of distortions in this economy).

The other set of variables that appear in (38) is related to the price and wage dispersion, which are caused by asynchronous nominal adjustment. These terms give rise to welfare losses even if the central bank manages to close the output gap completely because the lags in nominal adjustment generate inefficient dispersion of demands for individual goods and workers. Because goods and workers are imperfect substitutes, as can be seen from the aggregators (4) and (7), disproportional consumption of the individual goods results in spending more resources to produce a given level of the consumption bundle (and similarly for workers). To see this, notice that all the goods carry equal weight in the aggregator, hence the optimal allocation would require each firm to produce an equal amount of its own good.

4.1 Role of sectoral labour intensities

The first question addressed in this paper – Should the central bank respond to differences in labour intensities across sector? – can be answered by inspecting the social welfare

[2]The reader is kindly referred to Appendix for details of the derivation. The coefficients in (38) are defined as follows: $\gamma_y = U_C\overline{C}$, $\gamma_p = -V_L\overline{L}\,\frac{\epsilon\,\theta}{2(1-\epsilon)^2}$ and $\gamma_w = -V_L\overline{L}\,\frac{\eta\,\phi\,(1+\omega\,\phi)}{2\,(1-\eta)^2}$. Note that all these terms are positive and ρ is large than one, so that all terms in (38) enter with a negative sign.

[3]For an excellent exposition of the role played by each element of welfare functions of this sort see Woodford (2003).

function (38). The answer is "Yes", labour intensity affects the weights of some, not all though, of the sector-specific variables in the welfare function. The weight of sector j variables is inversely related to labour intensity $1-\alpha_j$, which means that the social planner can, other things equal, achieve a greater increase in the average utility by pursuing a policy that is biased towards lowering the variance in the capital-intensive sector. In order to understand the nature of the task faced by the central bank, let us inspect the origin of the sector-specific weights: the marginal product of labour.

As was already indicated in section 3.1, the slope (in absolute value) of the marginal product increases with capital intensity (falls with labour intensity), which implies that production of an additional unit of output makes capital-intensive firms hire more labour than labour-intensive firms. When demand for output increases, a capital-intensive firm needs to hire relatively more workers compared with the labour-intensive firm, where the marginal product is flatter, since the marginal product decreases faster with each additional worker hired.

This logic carries through also to the sectoral level. Rewriting the production function of an individual firm using the sectoral labour demand $L_j = \int L_j(f)\,df$ as $Y_j(f) = A_j (K_j/L_j)^{\alpha_j} L_j(f)$, it is possible to show that the sectoral labour demand is given by[4]

$$l_j = \frac{1}{1-\alpha_j}(y_j - a_j) + \frac{1}{2\,\theta\,(1-\alpha_j)} var_f y_j(f) \tag{39}$$

The first term on the right-hand side reflects the average amount of the labour index employed by firms in a sector, which is also the origin of the sectoral output-gap coefficients in the social welfare function, as shown in Appendix.

The other term on the right-hand side represents the extra labour cost caused by asynchronous price adjustment and imperfect substitutability among goods: if the demands for individual goods differ across firms then the total amount of output produced by individual firms (hence the total amount of labour) has to increase in order to meet a given demand for the sectoral output. The variance term can be directly related to price dispersion if one recalls that the demand for an individual good is given by (5), which implies that $var_f y_j(f) = \theta^2 var_f p_j(f)$. Erceg et al. (2000) then show that

$$E\, var_f p_j(f) = \frac{\epsilon}{(1-\epsilon)^2}\, var\,\pi_j$$

which gives rise to the price-inflation term in the social welfare function (38).

While the inefficiencies generated by contractual price setting depend on the relative labour-intensity of production, it is not the case with dispersion in individual labour hours

[4] This is equation (53) in Appendix.

that arises from wage rigidities. The key relationship in this regard is the log-linearised version of the sectoral labour aggregate (7), by means of which the fictitious labour agency aggregates the individual hours of work into the composite requested by firms. Relating it to the aggregate amount of hours supplied by households, $N_j = \int L_j(h)\, dh$, it is possible to derive the following relationship[5]

$$n_j = l_j + \frac{1}{2\,\phi}\, var_h l_j(h) \tag{40}$$

which shows that in order to produce a given amount of the labour index l_j households have to provide more labour hours in total when the individual labour supplies differ from each other. The non-degenerate dispersion in individual labour hours is, similarly to the price rigidities described above, caused by asynchronous wage adjustment and by the fact that individual workers are imperfect substitutes to each other (with greater elasticity of substitution between individual workers, ϕ, the inefficiencies decrease). With regard to the questions addressed in this paper, it is important to notice that these inefficiencies are independent of the labour-intensity because the way workers are assembled to the final labour composite is common to both sectors.

4.2 Role of aggregate output gap

The results obtained so far imply that the output gap and the price inflation in manufacturing receive a higher weight in the social welfare function (38) compared with their counterparts in services. However, the welfare function also shows that the central bank faces a trade-off when it tries to stabilise one sector (around it's efficient level) more intensively at the expense of greater volatility in the other sector. Mathematically, the trade-off is represented by the appearance of the aggregate output gap (square) in the welfare function (before taking unconditional expectations the variance term is in fact equal to $\hat{y}^2$). Since the weights of sectoral outputs in the aggregate output correspond to ψ_m and ψ_s and, most notably, are not affected by the respective labour intensities, minimizing this term would require to minimise both sectoral output gaps according to the weights they receive in the aggregate.

Intuitively, this term originates in households' relative willingness to adjust their labour supply and to substitute in consumption between the sectoral bundles. While it may be optimal for the central bank to push manufacturing always closer to its efficient level (to reduce the inefficiencies resulting from output variability), these efforts are limited on the consumers' side by the elasticity of their preferences. The latter is dictated by

[5] This is equation (48) in Appendix.

parameters ρ and ω, which are inversely related to elasticities of utility from consumption and labour, respectively. In particular, if ρ is relatively high (consumption is inelastic), consumers would prefer the central bank to respond equivalently to shock in both sectors to achieve a more balanced path of the aggregate consumption. Similarly, if ω is relatively low (labour supply is elastic), the fact that output dispersion requires greater shifts in the labour index hurts workers relatively less and thus the central bank is motivated to reduce output losses in both sectors in a more balanced way.

The desire to smooth the relative discrepancies between sectoral outputs is also magnified by the presence of wage rigidities. When workers are contractually committed to provide labour hours at the existing wage rate, adjustment to the efficient level of output lasts longer since the pull of the wealth effect, which would otherwise push demands in desired direction, is mitigated. Thus the welfare maximizing central bank has an additional incentive to curb the output gaps equally in both sectors to accelerate the aggregate output gap adjustment (by making the covariance between the output gaps more negative).

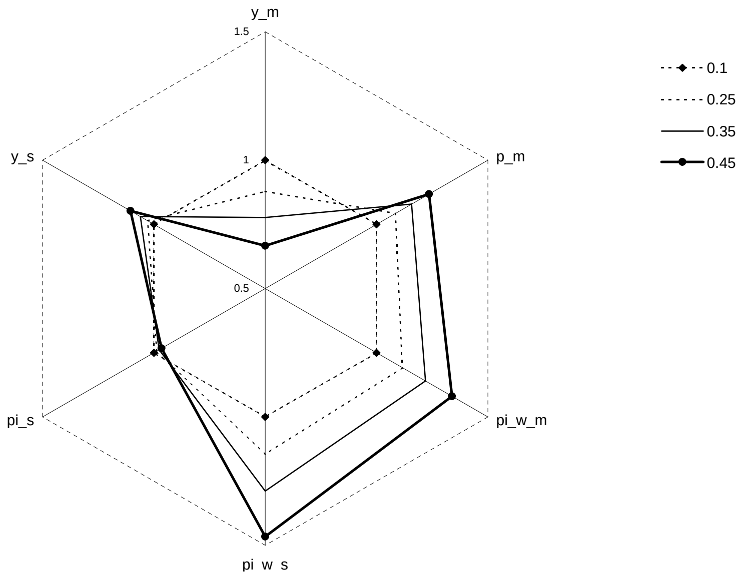
4.3 Optimal monetary policy

To solve for the optimal policy I follow the Lagrangean approach of Woodford (2003) which consists in maximizing the social welfare function (38), the exact form of which is given in Appendix, subject to the behavioural constraints presented in section 3.2. Next, I use the Anderson-Moore (1983) algorithm to map the system of the dynamic first-order conditions into an auto-regressive form, stationary of which then allows to iterate the system to obtain variances of all variables and, in the final step, to evaluate the social welfare function. Solving for an explicit policy rule, such as the Taylor rule, is not feasible here, similarly as in Benigno (2004).

The logic behind the optimal response of the central bank is demonstrated in Figures 1 and 2. The figures show the socially-optimal variances of sectoral variables (output gaps and inflation) that feature in the social welfare function. Figure 1 refers to the baseline scenario with an equal degree of price and wage rigidities, while Figure 2 shows the outcome with relatively low wage rigidity. All variances are expressed relative to the case when labour intensity is equal across sectors.

Both figures confirm the intuition described above: as capital intensity in manufacturing increases (labour intensity falls) it is optimal to reduce variance of the output gap in that sector (denoted as `y_m`) at the cost of higher variability of the output gap in services, `y_s`.

Figure 1: Variances of sectoral variables in the optimum: $\alpha_s = 0.10$, α_m varies



Note: The figure shows socially-optimal variances of sectoral output gaps `y`, rates of price inflation `pi` and wage inflation `pi_w` for different values of capital intensity in manufacturing, α_m. All variances are expressed relative to those achieved when $\alpha_m = \alpha_s = 0.1$.

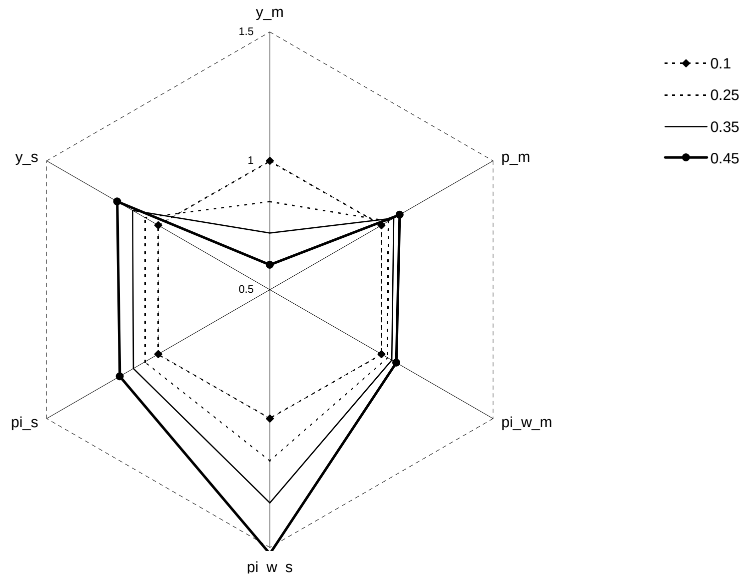
With relatively flexible wages, one can observe that the central bank is able to respond more emphatically to shocks in manufacturing to a much greater extent than the in the baseline scenario because the adjustment is faster overall. Hence, the trade-off between reducing the inefficient output dispersion in manufacturing and increasing the output gap overall (due to lower covariance between the two sectors) is less difficult.

As indicated above, the optimal plan is defined only implicitly as an auto-regressive stochastic process of the endogenous variables (and of the corresponding Lagrange multipliers). Such a prescription is, however, of limited value for the central bank that seeks to communicate its policy to general public in the hope to influence its inflation expectations. In the next section I will therefore analyze the optimal actions of the monetary authority assuming that it can commit itself to a credible inflation target.

4.4 Implications for inflation targeting

Having explored the socially optimal plan for this economy, we will now investigate how the optimal plan can be implemented by means of inflation targeting. In particular, the question is: What is the optimal inflation target in an economy with nominal frictions and two sectors that differ in the extent of capital and labour intensity?

Figure 2: Variances of sectoral variables in the optimum with less rigid wages



Note: The figure shows socially-optimal variances of sectoral output gaps `y`, rates of price inflation `pi` and wage inflation `pi_w` for different values of capital intensity in manufacturing, α_m. All variances are expressed relative to those achieved when $\alpha_m = \alpha_s = 0.1$.

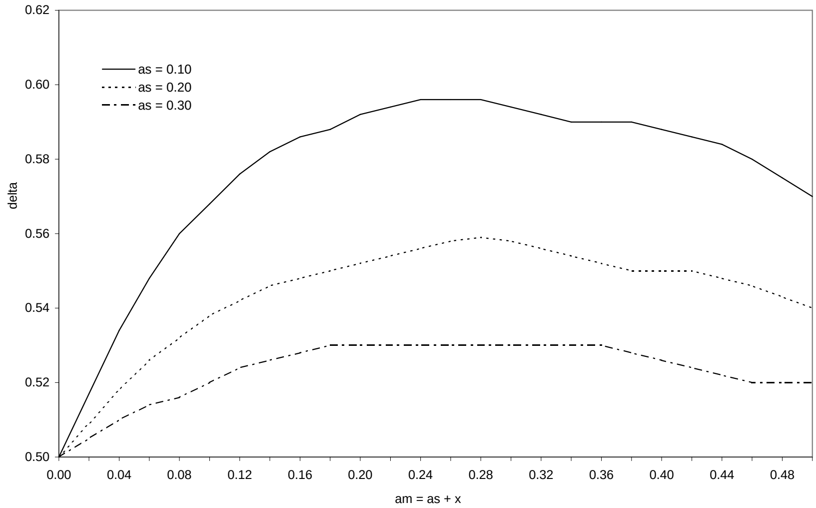
The choice of inflation targeting is grounded in the fact that rules based particularly on inflation do not require the knowledge of the efficient levels of output or relative prices, as would be the case if one wanted to target measures including also output *gaps*. In particular, we will inspect the performance of following inflation targeting rule

$$\delta\, \pi_m + (1 - \delta)\, \pi_s = 0$$

where we are interested in finding the optimal value of δ that allows the central bank to come as close as possible to the optimal plan. This rule can be understood as an implicit form of an interest-rate rule that requires the central bank to increase the interest rate when the weighted index of inflation increases above zero and vice versa, to decrease the nominal interest rate if the index drops below zero. To the extent that price inflation reflects output variability, as explained in section 4.1, it is also an indirect measure of shocks that hit the given sector. If prices and wages were fully flexible, the relative prices would immediately adjust to reflect the relative productivities in both sectors and the resulting demand shift would bring about the efficient structure (and level) of production.

With rigid prices (and wages), the central bank can, by adjusting the nominal interest rate, attempt to stimulate aggregate demand in order to mimic the efficient relative price adjustment so that the economy starts shifting toward the efficient level of output.

Figure 3: Optimal weight of manufacturing (δ) in the inflation target



Note: The figure shows the weight of inflation in the capital-intensive sector in the central bank's optimal inflation target. Optimal weights are reported for selected values of capital intensity in services `as` (different lines) and corresponding ranges of capital intensity in manufacturing `am` (along the horizontal axis).

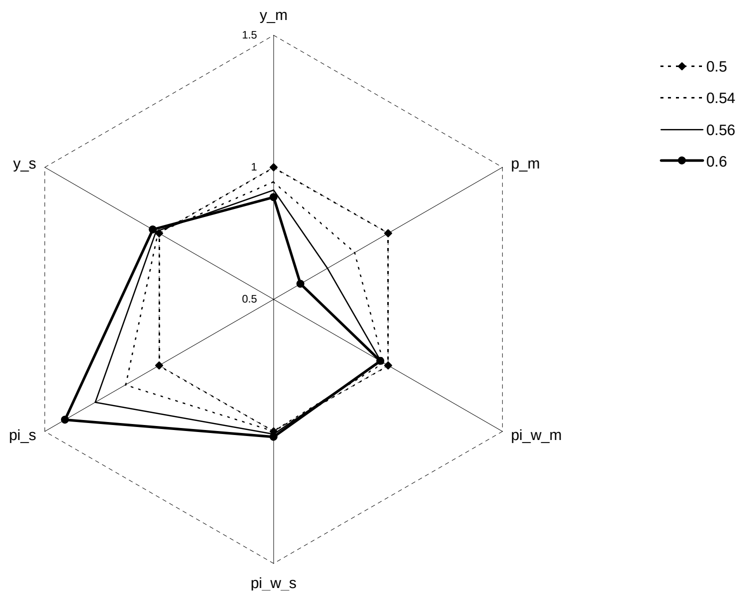
Furthermore, by adjusting the sectoral weights in the targeted inflation index it is able to systematically assign higher or lower priority to shocks (and output gaps) in a particular sector. Complete stabilisation of both output gaps in an economy, where both sectors exhibit nominal rigidities, is nevertheless infeasible (as shown in Benigno, 2004).

Figure 3 reports the optimal weights of manufacturing, δ, for selected values of capital intensity in services ($\alpha_s \in \{0.1, 0.2, 0.3\}$) and corresponding ranges of capital intensities in manufacturing defined as $\alpha_m \in (\alpha_s, \alpha_s + 0.5)$.

The key observation that emerges from the figure is that, as the capital intensity in manufacturing increases, the optimal inflation index assigns higher weight to manufacturing, which effectively means that the central bank prioritizes closing of the output gap in that sector. The intuition for this result goes back to the discussion in the section 4.1: the central bank tries to reduce inefficiencies resulting from output dispersion, the extent of which is greater in the relatively more capital-intensive sector. Note that if sectors do not differ in factor intensities, the figure shows that their respective weights in the inflation index exactly correspond to their weights in the consumption basket: $\psi_m = \psi_s = 0.5$ (for comparative purposes I will henceforward refer to such a policy as a *symmetric* targeting).

The outcome from following the optimal targeting policy on the variables included in the social welfare function is demonstrated in Figure 4, which confirms that, as the central bank starts moving away from the symmetric targeting rule ($\delta = \psi_m$) toward the

Figure 4: Elements of welfare on the way to the optimal policy



Note: The figure shows variances of sectoral output gaps `y`, rates of price inflation `pi` and wage inflation `pi_w` for different values of the share δ of the capital-intensive sector in central bank's inflation target. Capital intensities are $\alpha_s = 0.10$ and $\alpha_m = 0.25$, for which the optimal weight of manufacturing is 0.6. All variances are expressed relative to those achieved under a symmetric inflation target ($\delta = 0.5$).

optimal one, the output dispersion (i.e., price inflation) in manufacturing decreases and the output of this sectors follows its efficient path more closely. On the other hand, the output gap and, most significantly, the dispersion of individual outputs (price inflation) in services takes the opposite course.

Two other results follow from Figure 3. First, as the capital intensity of services increases as well, the extent of the optimal bias toward manufacturing decreases (this is represented by a shift to a lower curve in the figure)[6]. With greater use of capital in the economy overall, the marginal product of labour becomes steeper in both sectors and the preference to fight inefficiencies in only one of the sectors becomes less obvious (notice that the relationship between output dispersion and its cost in terms of the additional increase in the labour index in (39) is non-linear as well).

Finally, one can observe that the size of the optimal bias increases in a concave fashion and, in fact, as the *difference* in the capital intensity across sectors becomes large it eventually reverses its trend. This finding can be understood in the light of the central

[6]Mathematically, this can be seen from the relative weights of $\hat{y}_m$ and $\hat{y}_s$ in the objective function: $\psi_m \gamma_y \frac{1+\omega}{1-\alpha_m} / \psi_s \gamma_y \frac{1+\omega}{1-\alpha_s} = \frac{1-\alpha_s}{1-\alpha_m}$. Higher α_s decreases the relative weight of the output gap in manufacturing.

Table 1: Welfare improvement with the optimal inflation target

α_s	α_m	δ^*	Performance (%)
0.10	0.30	0.58	2.7
0.10	0.45	0.59	2.8
0.20	0.40	0.54	1.0
0.20	0.55	0.55	1.1
0.30	0.50	0.53	0.3
0.30	0.65	0.53	0.3

bank's trade-off indicated in section 4.2. As capital-intensity in manufacturing increases the central bank is tempted to close the manufacturing output gap with greater force than in services. However, by doing so the central bank allows the productivity shocks in services to take their natural course, which would eventually result in larger volatility of consumption and output overall. This can be tolerated only to the extent given by consumers' preferences. Should it become excessively volatile, the welfare maximizing central bank starts responding to the sectoral shocks in a more balanced way.

This trade-off also stands behind the poor performance of the inflation targeting rule in the quantitative sense. Table 1 compares the performance of the optimal inflation target to the symmetric one. The measure of performance compares the size of dead-weight losses eliminated under the targeting policy with optimal weights δ^* to the improvement achievable by implementing the fully optimal plan. Following Benigno (2004), it is constructed as follows

$$\frac{\widehat{SW}\left(\delta^*\right) - \widehat{SW}\left(\delta = 1/2\right)}{\widehat{SW}^* - \widehat{SW}\left(\delta = 1/2\right)}$$

It turns out that while targeting the optimal inflation measure represents an improvement compared to the symmetric target, the magnitude of welfare gains is rather small compared to that achievable under the fully optimal policy. This finding accords with conclusions of Erceg and Levin (2006), who inspect the performance of inflation targeting in a two-sectors economy, where sectors differ in the durability of their products. Neither in their model nor in the present paper is the inflation targeting policy able to achieve a substantial welfare improvement because it fails to move aggregate output sufficiently close to the efficient outcome.

5 Conclusions

The issue of optimal inflation targeting has received considerable interest in recent literature. Continuing in the research agenda of Rotemberg and Woodford (1997), who first implemented a micro-founded social welfare function to monetary theory, I study the optimal behaviour of the central bank in a two-sector economy with nominal rigidities, where sectors differ in the extent of their labour intensity. By deriving the social welfare function, I find that the optimal inflation to target in such an economy is systematically biased toward the capital-intensive sector.

The source of this bias is traced back to differences in the slope of the marginal product of labour in the two sectors. Nominal rigidities and limited substitutability across individual goods generate dispersion in the equilibrium amounts of output and labour, which is a source of welfare losses compared with the efficient outcome. Steeper marginal product of labour in manufacturing gives rise to higher efficiency costs of the fluctuations in the labour input in this sector. These findings complement those of Benigno (2004) and Erceg and Levin (2006) who conclude that the optimal measure of inflation to target should be biased toward the sector that exhibits a higher degree of nominal rigidities and a relatively more durable output, respectively. This paper focuses at highly realistic differences on the production side of the economy and examines their implications for the design of the optimal monetary policy.

References

Anderson, G., & Moore, G., 1983. An efficient procedure for solving linear perfect foresight models. Unpublished manuscript, Board of Governors of the Federal Reserve System.

Aoki, K., 2001. Optimal monetary responses to relative price changes. *Journal of Monetary Economics*, 48, 5580.

Benigno, P., 2004. Optimal monetary policy in a currency area. *Journal of International Economics*, 63, 293320.

Calvo, G., 1983. Staggered prices in a utility maximizing framework. *Journal of Monetary Economics*, 12, 383398.

Canzoneri, M., & Cumby, R., & Diba, B., 2005. Notes on models with calvo price and wage setting. Unpublished manuscript, Georgetown University, March 28.

Erceg, C.J., & Henderson, D.W., & Levin, A.T., 2000. Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46, 281313.

Erceg, C.J., & Levin, A.T., 2006. Optimal monetary policy with durable consumption goods. *Journal of Monetary Economics*, 53, 13411359.

Orphanides, A., & Wieland, V., 1998. Price stability and monetary policy effectiveness when nominal interest rates are bounded at zero. *Finance and Economics Discussion Paper, Board of Governors of the Federal Reserve System, Washington, DC.*, 98(35).

Rotemberg, J.J., & Woodford, M., 1997. An optimization-based econometric framework for the evaluation of monetary policy. In: B.S. Bernanke & J.J. Rotemberg (Eds.), *NBER Macroeconomic Annual 1997* (pp. 297 346). Cambridge: MIT Press.

Woodford, M., 1999. Optimal Monetary Policy Inertia. *NBER Working Paper*, 7261.

Woodford, M., 2003. *Interest and Prices: Foundations of a theory of monetary policy.* Princeton: Princeton University Press.

A Appendix

This section shows how to derive the social welfare function (38). It builds on strategies suggested in Aoki (2001), Benigno (2004) and Erceg et al. (2000), whose features the present model makes use of. Let us start with the average welfare of households in period t (compare to 37), where the time subscripts are dropped for ease of notation.

$$SW = 2\,U(C) + \int_0^1 V(L_m\,(h))\,dh + \int_0^1 V(L_s\,(h))\,dh \tag{41}$$

Implicit in the equation is the assumption of complete markets, which allows households to choose identical level of consumption in all states of nature. I will now take a second-order Taylor expansion of each element in the welfare criterion as is shown by the following preliminary example. The utility function can be approximated up to the second order by

$$U(C) = U(\overline{C}) + U_C\left(C - \overline{C}\right) + \frac{1}{2}U_{CC}\left(C - \overline{C}\right)^2 + o\left(\|\xi\|^3\right)$$

Noting that C can be written as $\overline{C}e^c$, where $c = ln\left(C/\overline{C}\right)$ and approximating it up to the second order by $\overline{C}\left(1 + c + \frac{1}{2}c^2\right) + o\left(\|\xi\|^3\right)$, we can rewrite (A) as

$$U(C) = U(\overline{C}) + U_C\overline{C}\left(c + \frac{1}{2}c^2\right) + \frac{1}{2}U_{CC}\overline{C}^2c^2 + o\left(\|\xi\|^3\right)$$

or, equivalently

$$U(C) = U_C\overline{C}\left(c + \frac{1}{2}c^2\right) + \frac{1}{2}U_{CC}\overline{C}^2c^2 + t.i.p. + o\left(\|\xi\|^3\right)$$

where the term $t.i.p.$ refers to terms independent of policy and the last term on the right-hand side includes all terms of order three and higher (henceforth I will neglect both of them). With these steps in mind and recalling that aggregate consumption ($C = Y/2$) is related to sectoral outputs according to (1), which I repeat here for convenience:

$$C = \frac{C_m^{\psi_m}C_s^{\psi_s}}{\left(\psi_m\right)^{\psi_m}\left(\psi_s\right)^{\psi_s}} \tag{42}$$

we can approximate the utility from consumption as follows.

$$\begin{aligned} U(Y/2) = {} & U_{Y_m}\overline{Y}_m\left(y_m + \frac{1}{2}y_m^2\right) + \frac{1}{2}U_{Y_mY_m}\overline{Y}_m^2y_m^2 \\ & + U_{Y_s}\overline{Y}_s\left(y_s + \frac{1}{2}y_s^2\right) + \frac{1}{2}U_{Y_sY_s}\overline{Y}_s^2y_s^2 \\ & + U_{Y_mY_s}\overline{Y}_m\overline{Y}_sy_my_s \end{aligned} \tag{43}$$

Furthermore, by differentiating (42) and setting $\rho = -U_{CC}\overline{C}/U_C$ one can prove the following relationships

$$\begin{aligned} U_{Y_j}\overline{Y}_j &= \psi_j U_C \overline{C} \\ U_{Y_m Y_m}\overline{Y}_m^2 &= -\psi_m U_C \overline{C}\left(\rho\psi_m + \psi_s\right) \\ U_{Y_s Y_s}\overline{Y}_s^2 &= -\psi_s U_C \overline{C}\left(\rho\psi_s + \psi_m\right) \\ U_{Y_m Y_s}\overline{Y}_m\overline{Y}_s &= \psi_m\psi_s U_C \overline{C}\left(1-\rho\right) \end{aligned} \tag{44}$$

that will be used shortly.

The second order approximation of each of the average labour supplies in (41) takes the form of

$$E_h V\left(L_j\left(h\right)\right) = V_L \overline{L}\left(E_h l_j\left(h\right) + \frac{1}{2}E_h l_j\left(h\right)^2\right) + \frac{1}{2}V_{LL}\overline{L}^2 E_h l_j\left(h\right)^2 \tag{45}$$

where E_h denotes an average over households in a sector. This step makes use of the fact that steady-state labour supply is equal in both sectors. To eliminate the quadratic terms in l_h we can use the definition of variance,

$$E_h l_j\left(h\right)^2 = var_h l_j\left(h\right) + \left(E_h l_j\left(h\right)\right)^2 \tag{46}$$

and the terms in expectations can be obtained from the equality of the aggregate labour hours supplied by workers to the employment agency and the aggregate demand by firms. The former is given by the labour index (7)

$$L_j \equiv \left[\int_0^1 L_j\left(h\right)^{\frac{\phi-1}{\phi}} dh\right]^{\frac{\phi}{\phi-1}}$$

which in the second order approximation reads as

$$l_j = E_h l_j\left(h\right) + \frac{1}{2}\frac{\phi-1}{\phi}var_h l_j\left(h\right) \tag{47}$$

Denoting N_j the total amount of hours actually supplied by households, $N_j = \int L_j\left(h\right)dh$, which can be approximated as $n_j = E_h l_j\left(h\right) + \frac{1}{2}var_h l_j\left(h\right)$, the above equation can be equivalently restated as

$$n_j = l_j + \frac{1}{2\,\phi}var_h l_j\left(h\right) \tag{48}$$

which is equation (40) in the main text. The left hand side of (47), the percentage change of the sectoral labour supply, must be in the equilibrium equal to the percentage change of the sectoral labour demand, which aggregates the labour demands of individual firms: $L_j = \int L_j\left(f\right)df$, or in the linear-quadratic approximation,

$$l_j = E_f l_j\left(f\right) + \frac{1}{2}var_f l_j\left(f\right) \tag{49}$$

Next, I make use of the production function to eliminate $E_f l_j(f)$. Recall that all firms in a sector employ identical capital-labour ratio, so that we can write $Y_j(f) = A_j (K_j/L_j)^{\alpha_j} L_j(f)$ or in the log-linear form: $y_j(f) = a_j - \alpha_j l_j + l_j(f)$ because the sectoral level of capital is fixed. From here we have the following relationships

$$E_f l_j(f) = E_f y_j(f) - a_j + \alpha_j l_j \tag{50}$$

$$var_f l_j(f) = var_f y_j(f) \tag{51}$$

Next, we employ the definition of the sectoral output, (4),

$$Y_j \equiv \left[\int Y_j(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}$$

which can be approximated by

$$y_j = E_f y_j(f) + \frac{1}{2}\frac{\theta-1}{\theta} var_f y_j(f) \tag{52}$$

Solving for $E_f y_j(f)$ from (52) and substituting it in (50), and then using it together with (51) in the sectoral labour demand equation, (49), we have equation (39), which appears in the main text:

$$l_j = \frac{1}{1-\alpha_j}(y_j - a_j) + \frac{1}{2\theta(1-\alpha_j)} var_f y_j(f) \tag{53}$$

Finally, equalizing the sectoral labour supply (47) and labour demand (53) we can solve for $E_h l_j(h)$

$$E_h l_j(h) = \frac{1}{1-\alpha_j}(y_j - a_j) + \frac{1}{2\theta(1-\alpha_j)} var_f y_j(f) - \frac{1}{2}\frac{\phi-1}{\phi} var_h l_j(h) \tag{54}$$

We are now ready to return to the average disutility from labour. Substitute first the first-order term of (54) in (46) and then use the result together with (54) again in (45) and rearrange the terms to get

$$\begin{aligned} E_h V(L_j(h)) = V_L \overline{L} \frac{y_j - a_j}{1-\alpha_j} + \frac{1}{2}\Bigg(& \frac{V_L \overline{L}}{\theta(1-\alpha_j)} var_f y_j(f) \\ & + \left(\frac{V_L \overline{L}}{\phi} + V_{LL}\overline{L}^2\right) var_h l_j(h) + \left(V_L\overline{L} + V_{LL}\overline{L}^2\right)\left(\frac{y_j - a_j}{1-\alpha_j}\right)^2 \Bigg) \end{aligned} \tag{55}$$

Before we proceed further it is useful to factor out $V_L \overline{L}$ from the second line and use the elasticity $\omega = V_{LL}\overline{L}/V_L$. Furthermore, the term $V_L\overline{L}/(1-\alpha_j)$ can be then replaced by $-2\,U_{Y_j}\overline{Y}_j$. This relationship follows from equalizing the supply of sectoral output (in the steady-state), as given in (22), to the demand for sectoral output from (2), where I made use of (20) to substitute for p_j. Solving for wages and substituting the resulting expression into the marginal rate of substitution, (16), gives the desired relationship.

The next step consists in employing the definitions from (44) to simplify (55) and (44) so that, after we substitute them back in (41), we have

$$\begin{aligned}\frac{SW}{U_C\overline{C}} &= \psi_m y_m^2 - \psi_m\left(\rho\psi_m + \psi_s\right) y_m^2 + \psi_s y_s^2 - \psi_s\left(\rho\psi_s + \psi_m\right) y_s^2 \\ &+ 2\psi_m\psi_s\left(1-\rho\right) y_m y_s + 2\psi_m a_m + 2\psi_s a_s \\ &- \psi_m\frac{1+\omega}{1-\alpha_m}\left(y_m - a_m\right)^2 - \psi_s\frac{1+\omega}{1-\alpha_s}\left(y_s - a_s\right)^2 \\ &- \frac{1}{\theta}\left(\psi_m var_f y_m\left(f\right) + \psi_s var_f y_s\left(f\right)\right) \\ &- \left(\frac{1}{\phi}+\omega\right)\left(\left(1-\alpha_m\right)\psi_m var_h l_m\left(h\right) + \left(1-\alpha_s\right)\psi_s var_h l_s\left(h\right)\right)\end{aligned} \tag{56}$$

With some algebra and using the definition of the aggregate output from (42), which in the log-linear form reads as $y = \psi_m y_m + \psi_s y_s$, the first three lines of (56) can be simplified as follows

$$\frac{SW_{1-3}}{U_C\overline{C}} = \left(1-\rho\right) y^2 - \psi_m\frac{1+\omega}{1-\alpha_m}\left(y_m - a_m\right)^2 - \psi_s\frac{1+\omega}{1-\alpha_s}\left(y_s - a_s\right)^2 \tag{57}$$

where I have neglected terms independent of output. Notice that an equivalent relationship can be obtained for the case without nominal rigidities. Subtracting the efficient solution from (57), expanding the squares, substituting for a_j from (26) and subsequently for a^w from (24), this equation simplifies in

$$\frac{\widehat{SW}_{1-3}}{U_C\overline{C}} = \left(1-\rho\right)\hat{y}^2 - \psi_m\frac{1+\omega}{1-\alpha_m}\hat{y}_m^2 - \psi_s\frac{1+\omega}{1-\alpha_s}\hat{y}_s^2 \tag{58}$$

where the terms with hat refer to output gaps, e.g. $\hat{y} = y - y^n$. Lastly, from the definition of the aggregate output and the demand curves (2) it is straightforward to derive the following relationships between the aggregate output gap and the relative price gap (defined as $\hat{p}_{rel} = p_{rel} - p_{rel}^n$): $\hat{y} = \hat{y}_m + \psi_s\hat{p}_{rel}$ and $\hat{y} = \hat{y}_s - \psi_m\hat{p}_{rel}$. Substituting these relationships in the equation above, it can be equivalently expressed as

$$\frac{\widehat{SW}_{1-3}}{U_C\overline{C}} = c_g\,\hat{y}^2 + 2\,\psi_m\psi_s\left(\frac{1+\omega}{1-\alpha_m} - \frac{1+\omega}{1-\alpha_s}\right)\hat{y}\,\hat{p}_{rel} - c_t\,\hat{p}_{rel}^2 \tag{59}$$

where

$$\begin{aligned} c_g &= \left(1-\rho\right) - \left(1+\omega\right)\left(\frac{\psi_m}{1-\alpha_m} + \frac{\psi_s}{1-\alpha_s}\right) \\ c_t &= \psi_m\psi_s\left(1+\omega\right)\left(\frac{\psi_m}{1-\alpha_s} + \frac{\psi_s}{1-\alpha_m}\right)\end{aligned} \tag{60}$$

Turning now to the last two rows in (56) we can rewrite the variance terms as follows. First, from the demand functions (5) and (8) we have that $var_h l_j\left(h\right) = \phi^2 var_h w_j\left(h\right)$ and $var_f y_j\left(f\right) = \theta^2 var_f p_j\left(f\right)$. Erceg et al. (2000) show that

$$E\,var_h w_j\left(h\right) = \frac{\eta}{\left(1-\eta\right)^2}\,var\,\pi_j^w \tag{61}$$

$$E\,var_f p_j\left(f\right) = \frac{\epsilon}{\left(1-\epsilon\right)^2}\,var\,\pi_j \tag{62}$$

Collecting the results of equation (58), taking unconditional expectations (the proof that $E\,\hat{y}^2 = var\,\hat{y}$ can be found in Erceg at al., 2000), and adding the variance terms from (61) and (62), the social welfare function becomes

$$\begin{aligned}\frac{E\,\widehat{SW}}{U_C\overline{C}} &= \left(1-\rho\right)var\,\hat{y} - \psi_m\frac{1+\omega}{1-\alpha_m}\,var\,\hat{y}_m - \psi_s\frac{1+\omega}{1-\alpha_s}\,var\,\hat{y}_s \\ &- \frac{\epsilon\theta}{\left(1-\epsilon\right)^2}\left(\psi_m\,var\,\pi_m + \psi_s\,var\,\pi_s\right) \\ &- \frac{\eta\phi\left(1+\omega\phi\right)}{\left(1-\eta\right)^2}\left(\psi_m\left(1-\alpha_m\right)\,var\,\pi_m^w + \psi_s\left(1-\alpha_s\right)\,var\,\pi_s^w\right)\end{aligned} \tag{63}$$

or alternatively, which is equation (38) in the main text,

$$\begin{aligned}E\,\widehat{SW} = \gamma_y\,\left(1-\rho\right)var\,\hat{y} - \psi_m\,\gamma_y\,\frac{1+\omega}{1-\alpha_m}\,var\,\hat{y}_m - \psi_s\,\gamma_y\,\frac{1+\omega}{1-\alpha_s}\,var\,\hat{y}_s \\ - \frac{\gamma_p}{1-\alpha_m}\,var\,\pi_m - \frac{\gamma_p}{1-\alpha_s}\,var\,\pi_s - \gamma_w\,var\,\pi_m^w - \gamma_w\,var\,\pi_s^w\end{aligned} \tag{64}$$

where the coefficients are defined as follows

$$\gamma_y = U_C\overline{C} \qquad \gamma_p = -V_L\overline{L}\,\frac{\epsilon\,\theta}{2\left(1-\epsilon\right)^2} \qquad \gamma_w = -V_L\overline{L}\,\frac{\eta\,\phi\,\left(1+\omega\,\phi\right)}{2\,\left(1-\eta\right)^2} \tag{65}$$